Competition Between Gravitational and Scalar Field Radiation

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ABSTRACT

Recent astrophysical observations have provided strong evidence that the present expansion of the universe is accelerating, powered by the energy density associated with a cosmological term. Assuming the latter to be not simply a constant term but a "quintessence" field, we study the radiation of quanta of such a "quintessence" field ("quintons") by binary systems of different types and compare intensities to those of standard tensor gravitational wave emission. We consider both the case in which the quintessence field varies only over cosmological distances and the case in which it is modified spatially by (strong) gravitational fields, a condition that results in bounds on the gradient of the scalar field. We show that, in both the first case and, because of a bound we derive from the Hulse-Taylor pulsar, in the second, there is not sufficient quinton radiation to affect expected LISA and LIGO gravity wave signals from binary systems. We show that, in the second case, the Large Hadron Collider is capable of setting a bound similar to that from the binary pulsar.

Subject headings: radiation mechanisms: general — gravitational waves — binary systems

1. Introduction

The past few years have witnessed a revolution in cosmology, a field that has been growing from data poor to data rich by leaps and bounds since the early 80's. This revolution followed the discovery, by the study of distant supernovae (Garnavich et al. 1998, Perlmutter et al. 1998), that the expansion of the universe is currently accelerating. A similar conclusion

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was reached in the "concordance analysis" of the cosmic microwave background (CMB) data by the WMAP collaboration (Bennett et al. 2003; Page et al. 2003) which concluded that the universe has a value of Ω, the ratio of its density to the critical density, consistent with $\Omega = 1$. In addition, the analysis of the WMAP data allowed an independent estimate of the contribution to this value by matter alone (including that of the "dark matter" which makes up most of the gravitationally bound matter); this was found not to exceed 30%, suggesting that a cosmological constant term could be responsible for the remainder, leading to an estimate $\Omega_\Lambda \simeq 0.7$.

The presence of a cosmological term of this magnitude has been the cause for great concern in the field: The "natural" value of such a term in the context of theories of the fundamental interactions would be huge, of order of the Planck energy density, the only "typical" value in a theory that contains a mass scale such as the Planck mass M_P (Carroll 2000; Peebles & Ratra 2003). The hope, then, was that a (hitherto unknown) conservation law would set its value to precisely zero, a value generally consistent with the pre-1998 data. The presence of a small, non-zero value for the cosmological term is then seen as another "fine tunning" problem in cosmology. Nevertheless, independently of the issue of the magnitude of the cosmological term, it is generally expected that this term does not represent a universal constant associated with the Lagrangian of gravitational interaction. Rather, it is thought that it likely varies like a field, whose value is constant on cosmological scales, while its magnitude is varying slowly in time. This field, being quite distinct from the other known fields and forms of matter has been given the name quintessence (Caldwell, Dave & Steinhardt 1998), borrowing the nomenclature from Aristotle's "fifth substance" that was supposedly involved in the make up of the universe.

Once, however, one decides that the term that drives the acceleration of the universe is not an *a priori* constant but a (scalar) field, one is immediately led to the notion that such a field obeys its own (scalar wave) equation and that it should be neither constant in time (a feature generally used in the literature) nor uniform on non-cosmological scales. As such it carries its own "kinetic" energy and potential energy which couple to the gravitational field thereby affecting both the metric around a bound object and also being affected by it, through the contributions of the metric to the covariant derivative.

Interestingly enough, an exact solution of the combined set of Einstein and zero-mass scalar field equations in the static spherically symmetric case has been derived independently several times in the literature, (e.g. Buchdahl 1959) including by one of us (Mannheim & Kazanas 1991). This last reference in particular, conjectured that the scalar field considered is none other than the Higgs field of high energy physics, which presumably is responsible for giving masses to particles through a spontaneous symmetry breaking process. This is rather

relevant in that our present study involves also the study of its effects in high energy collisions (of course, this field is expected to make the usual (huge) contribution to the cosmological constant, which must be somehow cancelled as discussed above; the effects discussed by Mannheim & Kazanas (1991) pertain only to the effects of the kinetic energy of the scalar field).

Motivated by the above considerations, we examine, in the present note, the case in which both the scalar and gravitational fields are space and time dependent, i.e. the case of quinton emission by their combined action. Such a study becomes imperative with the development of detectors of gravitational radiation that either currently are (TAMA) or about to become operational (LIGO), or are planned to be built in the not too distant future (LISA). Identifying gravitational waves in these facilities will depend on the comparison of the data to theoretically-derived, computer-generated, wave forms. These forms, and hence the interpretation of the data, could be impacted by competing processes not accounted for in template generation.

Due to the nature of our study, we have proceeded classically in the linearized regime with our main goal seeking conditions and physical situations for which radiation of scalars might become comparable to that of gravitational waves. To this end we explore both astrophysical systems, as well as systems in the laboratory (e.g. high energy collisions). We will also explore both the case in which the variation of the scalar field is limited to cosmological times and distances and the case in which the scalar field configuration is influenced by the presence of nearby (compact) objects.

Since we will be comparing scalar radiation with graviton radiation, we recall here (Shapiro & Teukolsky 1983) classical order of magnitude formulae for the latter. For a binary system, gravitational radiation luminosity is given in terms of the reduced and total masses, μ and M respectively, by the expression

$$L_{GW} = (32/5)G^4 M^3 \mu^2/(c^5 a^5) \tag{1}$$

with $M = M_1 + M_2$, $\mu^{-1} = M_1^{-1} + M_2^{-1}$, and $a = a_1 + a_2$ where M_i and a_i are the mass and distance from the center of mass (and origin) of body i ($i = 1, 2$) The rate at which the period, $\nu^{-1} = P = 2\pi/\omega$ ($\omega^2 a^3 = GM$), changes is given by

$$P^{-1} dP/dt = -(96G^3/5c^5)M^2\mu/a^4 \tag{2}$$

In §2 we set out the relevant equations and their linearized form while in §3 we examine the intensity of scalar field radiation resulting from the changing gravitational field (or metric) of a binary system and compare it to that of the gravitational radiation. We work, first, within the approximation that the (cosmological) scalar field is not affected by

the gravitational field of the binary system. In §4 we examine the competition between gravitational and scalar radiation taking into account the change in the scalar field due to the gravitational field of the binary using the solution of Mannheim & Kazanas (1991) and finally, in §5 we present and discuss our conclusions.

2. Basic Equations

Let Ψ denote a scalar field obeying the scalar field equation

$$\Box\Psi = (\partial_t^2 - \nabla^2)\Psi - \Gamma^\mu \partial_\mu \Psi = -m_{\rm eff}^2 \Psi = -\frac{\partial V(\Psi)}{\partial \Psi} \tag{3}$$

where the box operator denotes the covariant d' Alembertian, Γ^μ are the Christoffel symbols associated with the (in general) curved space into which Ψ operates and m is the (effective) mass of the (quinton) field Ψ. Our calculations are based on the approximation of first solving Equation (3) for the "semi-static" case and then computing the radiation field by approximating the right hand side by zero and using the Γ-term, with the semi-static quinton field, as the quinton radiation source.

Assuming space–time to be sufficiently close to flat, one can use a perturbative approach to calculate the components of the metric tenson $g^{\mu\nu}$ and then the Christoffel symbols. Thus, in the slow velocity regime, one can expand the metric tensor to powers of v/c, where v is the magnitude of the velocity of the matter components involved. In this approximation, to second order in v/c in g^{00} and to first order in v/c in g^{ii}, the space part of the metric tensor, the departure from flat is equal to twice the gravitational potential ϕ. In this approximation the diagonal metric tensor components and the square root of its determinant are (Weinberg 1972), remebering that $-g = \det(g_{\mu\nu})$,

$$g^{\mu\nu} = [g^{00}, g^{ii}] = [(-1+2\phi), (1+2\phi)], \text{ and } (-g)^{1/2} = \left[(1-2\phi)^2\right]^{1/2} = 1 - 2\phi + \mathcal{O}(\phi^2) \tag{4}$$

leading to

$$(-g)^{1/2} g^{\mu\nu} = \left[-1 + 4\phi + \mathcal{O}(\phi^2), 1 + \mathcal{O}(\phi^2)\right] \tag{5}$$

which then yields for the Christoffel symbols

$$\Gamma^\mu = (-g)^{-1/2} \partial_\nu \left[(-g)^{1/2} g^{\mu\nu}\right] = \left[\partial_0 \phi, \mathcal{O}(\phi^2)\right] \tag{6}$$

With the above expression for Γ^μ, the equation obeyed by the scalar field in curved space–time becomes

$$(\partial_t^2 - \nabla^2)\Psi - 4\, \partial_0\phi \partial_0 \Psi = -m^2 \Psi \simeq 0 \tag{7}$$

This equation is a wave equation with an inhomogeneous term. Its solution can be found in standard texts (e.g. Jackson 1962), i.e.

$$\Psi_R(x,t) = 4\int \frac{d^3x'}{\mid \vec{x} - \vec{x'} \mid} \partial_0 \Psi_0(\vec{x'}, t')\partial_0 \phi(\vec{x'}, t') \tag{8}$$

where Ψ_0 is the unperturbed (cosmologically varying only) scalar field at point x' and time t' and we are suppressing factors of c except when evaluating expressions numerically.

Considering the combined gravitational – scalar field variations appropriate for a binary system in circular orbit centered on the center of mass of the two bodies, the above equation reduces to

$$\Psi_R(x,t) = 4G(\partial_0\Psi_0)\int \frac{d^3x'}{|\vec{x'}-\vec{x}|}\left[M_1\frac{\vec{x'}-\vec{a_1}(t_1)}{|\vec{x'}-\vec{a_1}(t_1)|^3}\cdot\dot{\vec{a}}_1(t_1) + M_2\frac{\vec{x'}-\vec{a_2}(t_2)}{|\vec{x'}-\vec{a_2}(t_2)|^3}\cdot\dot{\vec{a}}_2(t_2)\right] \tag{9}$$

where

$$t' = t - |\vec{x} - \vec{x'}| \simeq t - x + \hat{x}\cdot\vec{x'} \tag{10}$$

and

$$t_i = t' - |\vec{x'} - \vec{a_i}(t_i)| \simeq t' - x' + \hat{x'}\cdot\vec{a_i}(t_i) \tag{11}$$

where $\vec{a_i}(t_i)$ is the "doubly retarded" vector connecting the center of mass of the system with body i. In other words t_i is the time at which a light signal from $\vec{a}_i(t_i)$ would have to leave in order to get to $\vec{x}$ at time t via $\vec{x'}$ at time $t' = t - |\vec{x} - \vec{x'}|$. We will also use $\partial_0\Psi = m_{\rm eff}\Psi_0$, with $m_{\rm eff}^2\Psi_0^2 \simeq U_{DE}$, where $U_{DE} = 10^3 eV/cm^3$ is the observed dark energy density The quinton (scalar field) at $\vec{x}$ is the result of coherent addition of the Ψ generated by the time dependent field at $\vec{x'}$ from the masses at $\vec{a}_i(t_i)$.

In the long wavelength approximation ($\omega^{-1} > a$), most of the contribution to the integral comes from distances less than $x' \sim \pi/\omega$.We take $|\vec{x} - \vec{x'}| \simeq x$ for the outer denominator. We use

$$\vec{a_i}\ (t_i) = \vec{a_i}\ (t' - |\hat{x'} - \vec{a_i}\ (t_i)|) \simeq \vec{a_i}\ e^{i\omega(t'-|\hat{x'}-\vec{a_i}(t_i)|)} \simeq \vec{a_i}\ e^{i\omega(t-x)}(1 + i\omega\hat{x'}\cdot\vec{a_i}) \tag{12}$$

In the same (long wavelength) approximation, we can ignore differences between the time dependence of a_1 and a_2 beyond that included in Equation (12). We now perform the time derivatives of the $\vec{a}$' s in Equation (9) making use of Equation (12). The result is

$$\Psi_R(\vec{x}, t) = 4GU_{DE}^{1/2}e^{2i\omega(t-x)}x^{-1}\int dx'd\Omega'\{\omega^2\,\hat{x'}\,[(\hat{x'}\cdot\vec{a_1})^2M_1 + (\hat{x'}\cdot\vec{a_2})^2M_2]\} \tag{13}$$

Other terms, at least in lowest order approximation, vanish and/or have the wrong time dependence. We make the approximations: (1) that the lower limits in the two x' integrals,

a_1 and a_2 are both approximately $a/2$, with $a = a_1 + a_2$ and (2) that the upper limits are of order π/ω past which the oscillating exponentials dampen any contribution. We use the fact that $M_1 a_1^2 + M_2 a_2^2 = \mu a^2$ to obtain

$$\Psi_R(x,t) = (16\pi^2/3x)e^{2i\omega(t-x)}\, G\, U_{DE}^{1/2} a^2 \omega\mu \tag{14}$$

With this result, the fact that the intensity L_Ψ of quinton radiation is $4\pi x^2 T^{0i}$, and the expression for the $0i$-component of the scalar field stress-energy tensor $T^{0i} = \partial_t\, \Psi_R\, \partial_r \Psi_R$, we obtain for the intensity

$$L_\Psi \simeq 10^4 U_{DE}\, G^2 \mu^2\, a^4 \omega^4 \tag{15}$$

where we have integrated over θ and averaged over θ_Ψ. This expression is applied to several specific cases in the next section. And, in the section after that, the analogue to this result, for the case in which the scalar field solution is modified by the gravitational field of the compact system, is derived and applied.

3. Slowly Varying Quintessence Field

We consider the case in which the scalar (quintessence) field (in section II) is not modified by the presence of strong gravitational fields. In the following section we consider the case studied by Mannheim & Kazanas (1991) in which the scalar field *is* modified by changes in the gravitational field.

We begin by applying Eq. (15) to a binary system in circular motion of radius a and Keplerian angular frequency ω. Using Kepler's law $\omega^2\, a^3 = GM$ we obtain in terms of $M_s = M/M_\odot, \mu_s = \mu/M_\odot$, and $a_{10} = a/10^{10}$ cm

$$L_\Psi = 8.7 \times 10^3 U_{DE} G^4 M^2 \mu^2/(c^7 a^2) \simeq 10^6 (\mu_s\, M_s/a_{10})^2 \ \text{ erg/s} \tag{16}$$

where we have inserted needed factors of c in Eq. (15) $[(U_{DE}c)(G^4/c^8)]$. Comparing with Eq.(1), $L_G \simeq 1.7 \times 10^{36} M_s^3 \mu_s^2/a_{10}^5$ erg/s, we see that

$$\frac{L_\Psi}{L_G} = 10^{-30}(\frac{a_{10}^3}{M_s}) \tag{17}$$

Thus quinton emission will dominate emission of gravitational radiation only for very large values of the orbit radius at which point they are both negligible.

Looking at larger mass objects, i.e. galaxies, clusters, superclusters etc... we can set $M \sim (4/3)\pi a^3 \rho_c \Omega_M$ or $M_s \sim 10^{-32} a_{10}^3$. Eq. (17) requires $a_{10} > 10^{11} M_S^{1/3}$ for quinton production to win. For two galaxies of $M \sim 10^{12}/M_\odot$, this gives 10^7 light years, somewhat

larger that the average galaxy separation. For a star in a galaxy circling the center of mass, L_Ψ is negligible compared to L_G.

Going the other way, we can look at small systems, asking how much energy is radiated in quintons in an excited state lifetime of an atom. Going back to Eq. (15) we insert $\omega a = v = \alpha c/n_B$, where n_B is the principal quantum number. Taking μ to be the mass of the electron, we see that

$$L_\Psi = 10^{-111}\, n_B^{-4} \ \ \text{eV/s} \tag{18}$$

We can also ask for the enchancement that would result in the case that there are large compact dimensions (Arkani-Hamed, Dimopoulos & Dvali et al., 1998) and the quintons can travel in the bulk. Following that reference for the case $n = 2$ of just two extra compact dimensions, we would get from Eq. (9), for an atom ($a \sim 10^{-8}$),

$$\frac{d^3x'}{x'^2} \rightarrow \frac{d^5x'}{x'^4} \sim \left(\frac{\text{mm}}{a}\right)^2 \tag{19}$$

The result would be to modify Eq.(18) by a factor of $\sim 10^{28}$ which does not appear enough to make it of experimental interest. There is a larger effect if we consider a smaller (nuclear) system such as $\alpha-$decay of a long-lived isotope. In that case, a 10 MeV $\alpha-$particle could give $(\omega\, a)^2 \sim 10^{-2}c^2$ so that Eq. (16) would be

$$L_\Psi = 10^4(10^3 eV/cm^3)c\left(\frac{m_\alpha}{m_{Pl}}\right)^2\left(\frac{\hbar c}{m_{Pl}}\right)^2\left(\frac{\omega a}{c}\right)^4 (1\text{mm}/5\cdot 10^{-13}\text{cm})^4 \sim 10^{-45} \ \ \text{eV/s} \tag{20}$$

Thus, even a mole (10^{24}) of an isotope with a billion year half life would have less than an electron volt of energy loss into quinton radiation. Finally, we turn to accelerator production, $p-p$ collisions at the LHC. We take $a \sim \sigma^{1/2} \sim 10^{-13}$ cm and compute the energy radiated into quintons in a collision.

$$\Delta E = L_\Psi \delta t \tag{21}$$

with $\delta t = a/c$ and

$$L_\Psi = 8.7 \times 10^3 (U_{DE}G^2)(\mu^2/\hbar)(a\omega/c)^4(1mm/a)^4 \tag{22}$$

where we have, again, assumed two extra compact dimensions. We have also assumed $a\omega = c$, although it is possible that, in a quantum treatment, we might have $a\omega \rightarrow aE$ giving a much larger result. We have, of course, set $\mu = E$. Again the result is small: $\Delta E \sim 10^{-62} GeV$. Thus, based on a quinton field varying only over cosmological times, quinton radiation does not approach gravitational radiation for any of the 3 cases considered in atomic and nuclear transitions and high energy collisions, nor in the astrophysical binary systems considered.

4. Quintessence Field Varying in Strong Gravitational Field

We turn now to the possibility that the scalar field is modified in the presence of gravitational fields. Specifically, we address the cases of Section III in light of the results of the work of Mannheim & Kazanas (1991). These authors considered Eq. (3) written in the form

$$\frac{1}{(-g)^{1/2}}\partial_\mu[(-g)^{1/2}\partial^\mu\Psi] = \frac{\partial V}{\partial\Psi} \simeq 0 \tag{23}$$

along with Einstein's equations written in the form

$$\frac{1}{8\pi G}G^{\mu\nu} - \partial^\mu\Psi\partial^\nu\Psi + \frac{1}{2}g^{\mu\nu}\partial^\alpha\Psi\partial_\alpha\Psi = -g^{\mu\nu}V(\Psi) \tag{24}$$

They find, as did Buchdahl (1959), closed form solutions for the case $V(\Psi) = 0$. With $V(\Psi) \simeq m_{\rm eff}^2\Psi^2$ and $m_{\rm eff} \sim 10^{-33}$ eV, as demanded by the dark energy observations, their two solutions should be good approximations. One is that Ψ is a constant, unaffected by gravitational fields. This, of course, is just the case considered above. The second solution, of the coupled equations, is

$$ds^2 = -H(\rho)dt^2 + J(\rho)[d\rho^2 + \rho^2 d\Omega] \tag{25}$$

The functions in the above equation are

$$\Psi(\rho) = \frac{K}{2r_0}ln\left(\frac{\rho - r_0}{\rho + r_0}\right) + constant \tag{26}$$

$$H(\rho) = \left(\frac{\rho - r_0}{\rho + r_0}\right)^{-d/2r_0} \tag{27}$$

$$J(\rho) = \left(1 - \frac{r_0^2}{\rho^2}\right)^2\left(\frac{\rho - r_0}{\rho + r_0}\right)^{-d/2r_0} \tag{28}$$

where

$$d = 2MG = 4(r_0^2 - \pi G K^2)^{1/2} \tag{29}$$

and it can be shown (Mannheim & Kazanas 1991) that r_0 is restricted to the region

$$MG/2 \leq r_0 \leq MG, \tag{30}$$

indicating that r_0 is of the same order of magnitude as the Schwarzschild radius. We set $\pi K^2 = GM^2\beta^2$, $r_0 = MG\gamma$ with $0 \leq \beta \leq 1$ and $1/2 \leq \gamma \leq 1$. We assume that the dimensionless quantities, β and γ, are independent of the gravitational field, that is, like Newton's constant, are the same for all masses.

Using the above equations we compute $\Gamma^\mu, \Psi_R, L_\Psi$ and L_Ψ/L_{GW}. We first obtain

$$g^{-1/2}\partial_\rho(g^{1/2}g^{\rho\rho}) \;=\; \Gamma^\rho \tag{31}$$

$$= \frac{2}{\rho^3}\left(1-\frac{r_0^2}{\rho^2}\right)^2\left(\frac{\rho-r_0}{\rho+r_0}\right)^{-1/2\gamma}(\rho^2-6r_0^2-\frac{1}{\gamma}r_0\rho) \tag{32}$$

$$\simeq \frac{2}{\rho} \;\text{ for }\; \rho > a \gg r_0 \tag{33}$$

Similarly, we have $\Gamma^0 = (2/\rho)\partial_0\rho,\ \partial_\rho\Psi \simeq K/\rho^2$, and $\partial_0\Psi \simeq (K/\rho^2)\partial_0\rho$.

We rewrite the solution of the wave equation (3) as

$$\Psi_{MK}(\vec{x},t) = x^{-1}\int d^3x'\Gamma^\mu(\vec{x'},t')\partial_\mu\Psi_0 \tag{34}$$

for each of the two bodies in the binary system separately. Using $\vec{\rho}{=}x' - \vec{a}_i(t_i)$, we will add the two contributions to Ψ_{MK}. The time and ρ components give much different results for Ψ_{MK} and L_{MK}. For the time component, $\partial_0\vec{\rho}$ is simply $\partial_0\vec{a}$ and the leading contribution is

$$\Psi_{MK,0} = x^{-1}\int d^3x'(4i\omega\,\vec{a_1}\cdot\hat{x}')(2i\omega\,\vec{a_1}\cdot\hat{x}')K_1/x'^3 = (32\pi/3x)e^{2i\omega(t-x)}K_1a_1^2\omega^2 ln(1/a\omega)+(1\to 2) \tag{35}$$

which gives, after using $\pi(K_1a_1^2+K_2a_2^2)^2 = \beta^2G\mu^2a^4$,

$$L_{MK,0} = (64\pi/3)^2\beta^2(G\mu^2)(a\omega)^4\omega^2 ln^2(a\omega/c) \tag{36}$$

The ρ contribution is given by

$$\Psi_{MK,\rho} = (2K_1/x)\int d^3x'/\mid\vec{x'}-\vec{a_1}\mid^3 + (1\to 2) \tag{37}$$

We expand the denominator(s) in powers of the a_i; noting that the term of first order in a_i vanishes after the angular integration, we are left with terms of order 2. The result is

$$\Psi_{MK,\rho} = (6K_1/x)\int d^3x'[4(\hat{x'}\cdot\vec{a_1})^2 - a_2^2]/x'^5 + (1\to 2) = 2\pi(K_1+K_2) \tag{38}$$

from which we get

$$L_{MK,\rho} = 2(4\pi)^2G\mu^2\omega^2\beta^2 \tag{39}$$

Here, we have approximated $\pi(K_1^2+K_2^2)$ by its equal mass value.

Clearly, the relative factor of $(a\omega/c)^4$ between $L_{MK,0}$ and $L_{MK,\rho}$ means that the latter will be more important for astrophysical objects and as well as atomic and nuclear ones,

while the former will dominate in high energy physics. Turning first to astrophysics, one may note

$$\frac{L_{MK,\rho}}{L_{GW}} \simeq \frac{16\pi\beta^2 G\mu^2\omega^2}{\frac{64}{5}G\mu^2 a^4\omega^6} = \frac{5\beta^2/4}{a^4\omega^4} \simeq \frac{\beta^2}{(v/c)^4} \tag{40}$$

where v is the velocity of the lighter member in the non-equal mass case. We note the similarity between Eq.(39) and L_{GW}. This might be expected since there is no additional mass scale, such as the U_{DE} in section II which enters in forming Ψ_R. .

We can now estimate an upper bound on β from the Hulse – Taylor pulsar which has an orbital period of 28,000 sec and the fact that the rate change of its orbital period agrees to better than one percent with the prediction of General Relativity. From the fact that $L_{MK}/L_{GW} < 0.01$ we obtain that $\beta^2 < 2 \times 10^{-3}(v/c)^4 \simeq 10^{-15}$. While this appears a stringent bound on the constant K, understanding its full implications awaits simultaneous solution of Equations (23, 24) in the interior region as well as the exterior region which should permit evaluating K and r_0 in terms of the interior mass distribution.

Turning to the high energy case, we compute the energy that would be lost into quinton radiation in proton-proton collisions. $L_{MK,0}$ of Equation (36) above dominates. In it, we take μ to be 7 TeV as with the LHC in section 3 and ω to be c/a, while recognizing that it might be larger in a quantum treatment. Again, we multiply L by a/c to obtain the energy radiated during the encounter. We include enhancement from two extra compact dimensions. We ignore the logarithm. These give

$$\Delta E = (64\pi/3)^2(E/m_{Pl})^2\beta^2(\omega^2 a/c)(1mm/a)^4 = 10^{18}\beta^2 GeV \tag{41}$$

This implies that, if significant unexplained energy loss in $p-p$ collisions at the LHC is not found, a bound on β^2 slightly better than that above from the Hulse-Taylor pulsar can be inferred for models with compact extra dimensions. It should also be possible to infer such a bound from cosmic ray data from the Auger project (www.auger.org) which will study p-p scattering at about 30 times LHC energies (at the center of Mmss). One might even infer from the existence of ultra high energy cosmic ray observations that proton-proton interactions do not lose large amounts of energy into unobserved particles, implying a bound on β^2 of about 10^{-21}. On the other hand, the cosmic ray spectrum does exhibit a feature known as the "knee" at an energy $E_k \simeq 10^{15.5}$ eV which corresponds to roughly 1 TeV at the center of mass. This is a steepening in the slope cosmic ray spectrum by 0.3 - 0.4 over half a decade in energy. The very limited energy range over which this change in the cosmic ray spectral index occurs essentially precludes its explanation as simply a cosmic ray propagation effect. In fact Kazanas & Nicolaidis (2003) suggested that this feature heralds

the emergence of physics beyond the Standard Model. In particular, these authors have argued that such a feature is the result of energy lost in cosmic ray collisions to gravitons, as suggested by the theories of extra large dimensions which presumably have the graviational constant increase with energy to that of strong interactions at energies ~ 1 TeV. It would not be in conflict with our Hulse-Taylor pulsar bound, if quinton emission were also to contribute to this cosmic ray feature.

5. Summary

General relativity requires that any quintessence field couple, through the covariant derivative, to the gravitational field. Thus, any time varying gravitational field will produce quintons. The rate of production, however, will be proportional to the square of the derivative of the quintessence field. We have evaluated that rate, in a classical approximation, in Section II, for the case in which the field only varies over cosmological times. We applied the results to a variety of binary systems in Section III, but found no cases of interest in which energy loss through quintons dominated energy loss through gravitons.

In Section IV we turned to the perhaps more realistic case in which the space variation of the quintessence field is affected by the presence of a gravitational field. We worked with the exact (exterior) solution for the massless case as written down by Mannheim and Kazanas (1991). The result, in that case, was quinton production dominated by different components for low velocities of the binary system than for high. We were able to bound an integration constant in the solution cited by requiring that quinton emission from the Hulse-Taylor binary pulsar system represents less than one percent of the energy loss. That limit on the parameter was sufficient to make it difficult to identify observable effects in the astrophysical phenomena considered. We were also able to show that, with the assumption of large, compact extra dimensions similar and stronger bounds could be derived from data, when available, from the LHC and project Auger (and perhaps from current cosmic ray data showing the existence of ultra high energy cosmic rays).

In summary, it appears that quinton emission is unlikely to be of importance in interpreting signals received by LIGO or LISA, or in laboratory experiments if the quinton field has no coupling to matter beyond the indirect coupling provided by the covariant derivative or if large compact extra dimensions do not exist.

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REFERENCES

Arkani-Hamed, N., Dimopoulos, S. & Dvali, G. 1998, Phys. Lett., 429, 263

Bennett, C. L. et al. 2003, ApJS, 148, 1

Buchdahl, H. A. 1959, Phys. Rev., 115, 1325

Carroll, S. M. 2000, in "Living Reviews in Relativity", Pub. No 2000-1

Caldwell, R. R., Dave, R. & Steinhardt P. J. 1998, Phys. Rev. Lett., 80, 582

Garnavich, P. M. 1998, ApJ, 493 L53

Jackson, J. D., 1962, "Classical Electrodynamics", John Wiley & Sons, New York.

Kazanas, D. & Nicolaidis, A. 2003, Gen. Rel. Grav., 35, 1117

Mannheim, P. D. & Kazanas, D. 1991, Astrophys. Sp. Sci., 579, L67

Page, L. et al. 2003, ApJS, 148, 283

Peebles, P. J. E. and Ratra, B. 2003, Rev. Mod. Phys., 75, 559

Perlmutter, S. et al. 1998, Nature, 391, 51

Shapiro, S. L. & Teukolsky, S. H. 1983 "White Dwarfs, Neutron Stars and Black Holes", John Wiley, New York

Weinberg, S. 1972, "Gravitation & Cosmology", John Wiley, New York